April 21

Announcements
- On quizzes, show your work!
- Adam office hrs every Thurs 11:30-12:20

<u>Defn</u> Given $K \subset L$, an <u>intermediate field extension</u> is a field F such that $K \subset F \subset L$.

Ex: $\mathbb{Q} \subset \mathbb{Q}(\sqrt{2}) \subset \mathbb{Q}(\sqrt{2}, \sqrt{3})$

- F need not be simple
- Allow $F = K$ or $F = L$.

How to compute degree $|L:K|$?

- exhibit a basis of L/K
- If you know $L = K(\alpha)$, then $|L:K| = $ deg min poly of α
- $K \subset F \subset L$ $|L:K| = |L:F||F:K|$

Where we are headed?

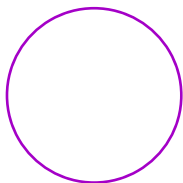Next topic: splitting fields, normal & separable field ext, finite fields, & then define Galois groups. —

Today & Friday   (detour)

Ruler & compass constructions

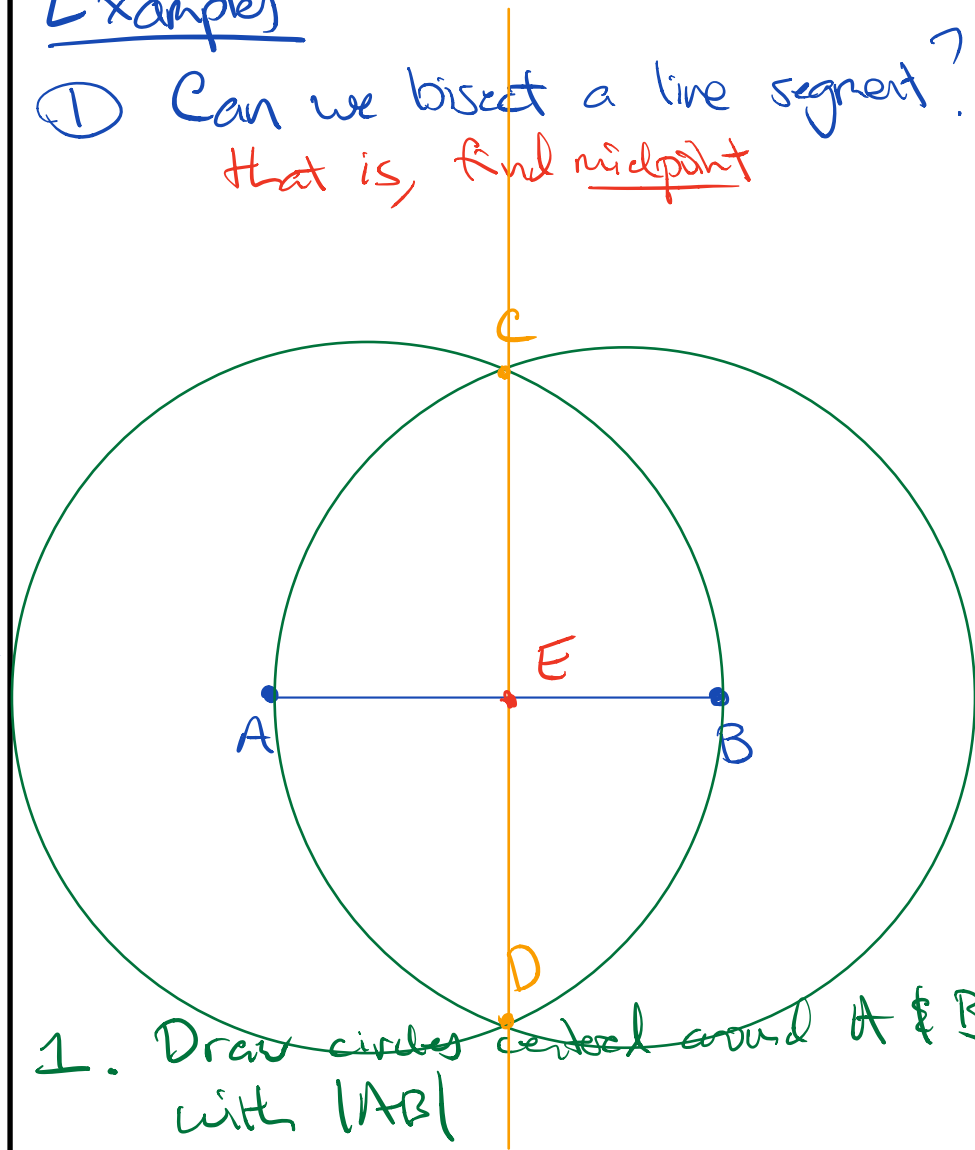PLATO: The only perfect geometric shapes are lines & circles.

ruler: draw lines through 2 pt

compass
draw a circle centered around any pt with a given radius

Examples
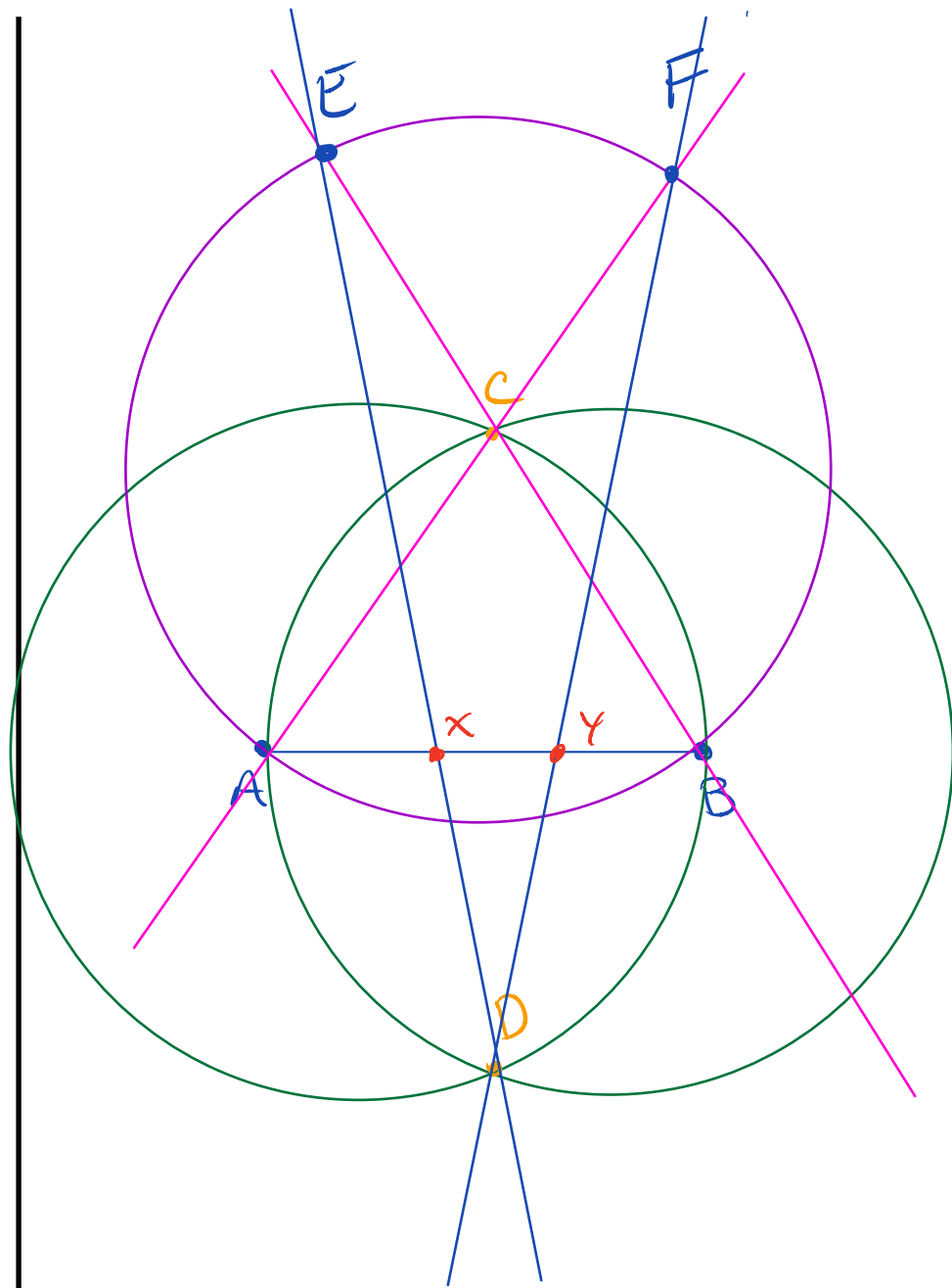
① Can we bisect a line segment?
that is, find midpoint

1. Draw circles centered around A & B with |AB|

2. Draw line segment CD

3. Midpoint is E

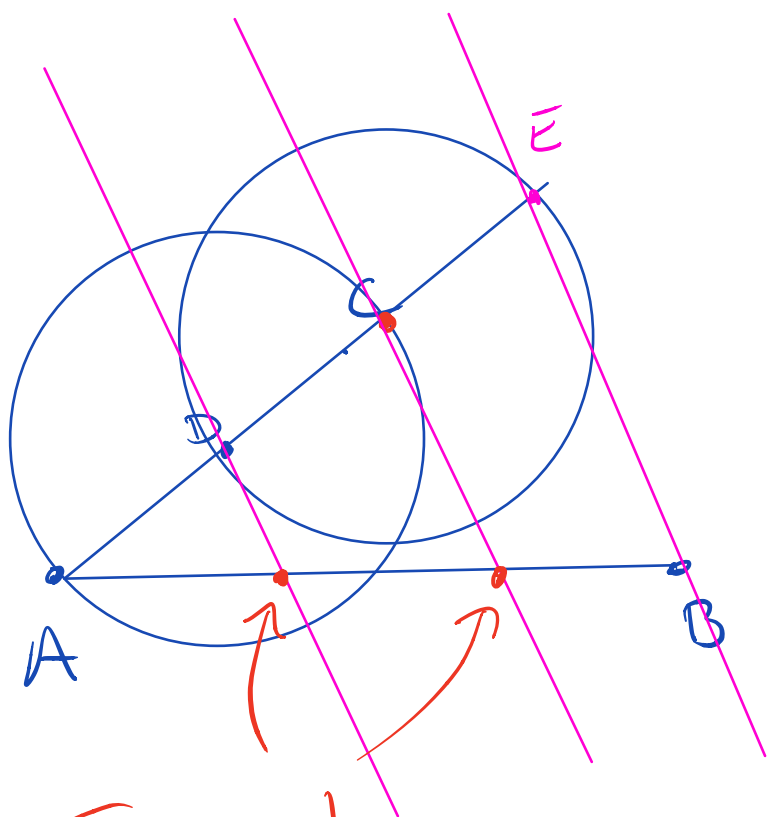**Ex 2** Can we trisect a line segment?

A   x   y   B

Goal: Find pts x & y that divide line segment into 3 equal parts

1. Draw circles centered around A & B with $|AB|$

2. Draw circle centered around **C** with radius $|AC|$ $(= |AB|)$

3. Draw lines AC & BC

4. Draw lines ED & FD

5. x & y trisect line!

Exer: Prove this works

E

C

D

A

B

Trisect!

Formalizing this concept

Given points $P_1, \ldots, P_n \in \mathbb{R}^2$

Two operations

① Ruler : Through any 2 points P_i, P_j, can draw a straight line

② Compass : Given a point P_i and another point P_j, can draw a circle centered around P_i of radius $|P_i P_j|$

→ New points of intersection that are added to $P_1, \ldots P_n$

Defn Given $P_1, \ldots, P_n \in \mathbb{R}^2$

• Say a point $r \in \mathbb{R}^2$ is constructible in one step from $P_1, \ldots P_n$ if r is the intersection point of two distinct lines, two distinct circles, or a circle & line.

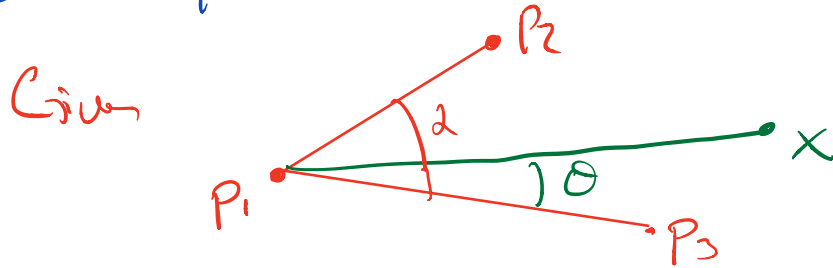• Say $r \in \mathbb{R}^2$ is constructible from $P_1, \ldots P_n$ if $\exists r_1, \ldots, r_s$ where each r_i is constructible in one step from $P_1, \ldots, P_n, r_1, \ldots, r_{i-1}$ and $r_s = r$.

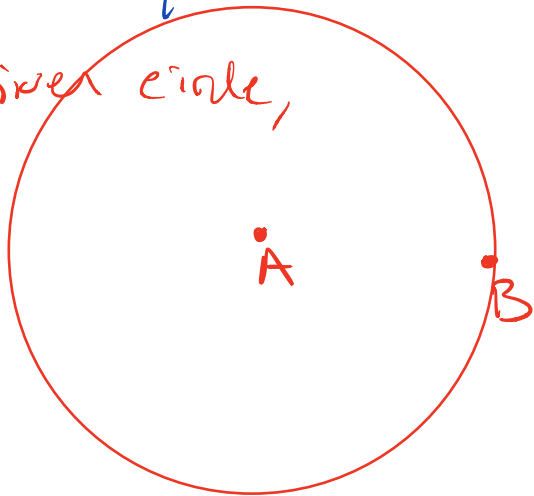Greeks considered these problems

① Can you trisect an angle?
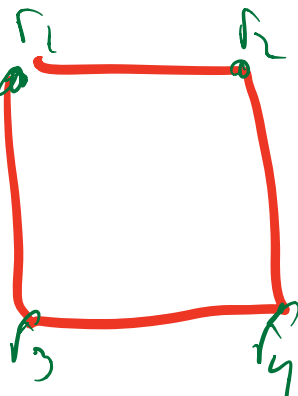
Given

Find x s.t. $\theta = \frac{1}{3}\alpha$

(Is x constructible from P_1, P_2, P_3?)
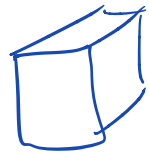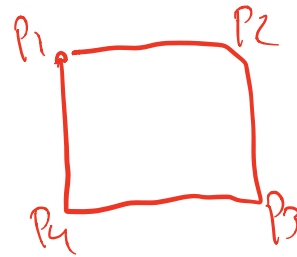
② Can you square the circle?

Given circle,

r_i's should be constructible from A,B
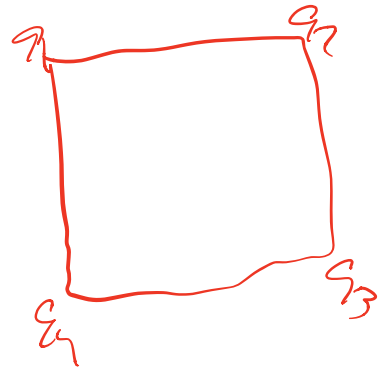
can you find square with same area?

③ Given a cube

can find another cube whose volume is twice original?

Given

, can you

find another square

$$|q_1 q_2|^3 = 2|p_1 p_2|^3$$